Fiscal-Monetary Interactions and the FTPL:

“Some Unpleasant Monetarist Arithmetic”
Sargent and Wallace (1981)

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Motivation

In his AEA presidential address in 1968, Friedman said that monetary policy cannot affect

- output and unemployment
- the real rate of return on securities

Sargent and Wallace (1981) show that, under some condition, mp cannot even affect inflation!

Environment

- OLG model
- population and income grow at constant rate n
- storage technology with real return $r > n$
- upper bound on per-capita real debt stock
- "monetarist" economy: monetary base connected to P_t via quantity-theory money demand

Model

At period t, $N_{1,t}$ poor agents and $N_{2,t}$ rich agents. $N_{1,t} + N_{2,t} = N_t$. All Ns grow at n

Poor have endowments α_1, α_2, cannot use storage or bonds, save through money

Rich have endowments $\beta, 0$, will save in highest return asset

Problem of class j, generic savings W and RoR ρ

$$\begin{aligned} \max_{c^y, c^o, W} \quad & c^y c^o \\ \text{s.t.} \quad & c^y = e^y - W \\ & c^o = e^o + W\rho \end{aligned}$$

Savings function $W = \frac{1}{2}\left(e^y - \frac{e^o}{\rho}\right)$

Focus on case where $P_t / P_{t+1} < 1 + r$, so only the poor hold money

Asset demand

Savings function $W = \frac{1}{2}\left(e^y - \frac{e^o}{\rho}\right)$
Focus on case where $P_t/P_{t+1} < R$, so only the poor hold money

Money demand by the poor + money market clearing

$$\frac{M_t}{P_t} = N_{1,t}\frac{1}{2}\left(\alpha_1 - \alpha_2\frac{P_{t+1}}{P_t}\right)$$

For simplicity, for now assume $\alpha_2 = 0 \Rightarrow$ pure quantity theory, money stock determines P_t

The rich save $\beta/2$. If bonds yield r, then total bond demand $\leq N_{2,t}\beta/2$
$\Rightarrow$ real bonds per capita are capped at $\frac{N_{2,t}}{N_t}\frac{\beta}{2}$

Government

Govt budget

$$d_t = b_t - b_{t-1}(1+r) + \frac{M_t - M_{t-1}}{P_t}$$

in per-capita terms $\tilde{x}_t = x_t / N_t$

$$\begin{aligned}
\tilde{d}_t &= \tilde{b}_t - \tilde{b}_{t-1}\frac{1+r}{1+n} + \tilde{m}_t - \frac{\tilde{m}_{t-1}}{(1+n)}\frac{P_{t-1}}{P_t} \\
&= \tilde{b}_t - \tilde{b}_{t-1}\frac{1+r}{1+n} + \frac{\alpha_1}{2}\left(1 - \frac{1}{(1+n)}\frac{P_{t-1}}{P_t}\right)
\end{aligned}$$

Let T be the time at which per-capita debt hits the debt ceiling

Monetary policy

- for $t < T$, constant money growth $M_{t+1} = M_t(1+\theta)$. Inflation is $\dfrac{P_{t+1}}{P_t} = \dfrac{1+\theta}{1+n}$
- for $t \geq T$, M_t set to keep $\tilde{b}_t = \tilde{b}_T$

Inflation before vs after T

Step 1: study how inflation after T ("future") depends on $\tilde{b}_T$
Step 2: study how $\tilde{b}_T$ depends on inflation/money growth/monetary policy before T ("now")

Step 1: for $t \geq T$

$$\tilde{b}_T(\theta)\frac{r-n}{1+n} + \tilde{d}_t = \frac{\alpha_1}{2}\left(1 - \frac{1}{(1+n)}\frac{P_{t-1}}{P_t}\right)$$

since $r > n$, then higher $b_T(\theta)$ requires higher inflation

- pc deficit = pc primary deficit ($\tilde{d}_t$) + pc debt service ($\tilde{b}_T(\theta)\frac{r-n}{1+n}$)
- higher terminal debt $\rightarrow$ higher terminal deficit to be financed $\rightarrow$ higher future seigniorage revenues needed
- future inflation is increasing in the future primary deficit

Inflation before vs after T

Step 2: for $t < T$, how does θ affect $\tilde{b}_T(\theta)$?

Take the period-by-period govt BC

$$\tilde{d}_t = \tilde{b}_t - \tilde{b}_{t-1}\frac{1+r}{1+n} + \frac{\alpha_1}{2}\frac{\theta}{1+\theta}$$

Iterate govt BC forward from the initial period $(t = 0)$

$$\tilde{b}_T(\theta) = \sum_{s=0}^{T-1}\left(\frac{1+r}{1+n}\right)^s\left(\tilde{d}_{T-s} - \frac{\alpha_1}{2}\frac{\theta}{1+\theta}\right) + \left(\frac{1+r}{1+n}\right)^T\tilde{b}_0$$

so a lower θ implies a higher $\tilde{b}_T(\theta)$

- $\tilde{b}_T$ depends on deficits from $t = 0$ to $t = T$
- tighter money now $\rightarrow$ lower seigniorage/inflation now $\rightarrow$ higher pc debt at T $\rightarrow$ as we saw, higher future inflation!
- what is happening is monetary dominance now and fiscal dominance in the future

Inflation before vs after T

Can tight money now mean not only higher future inflation, but also higher inflation now?

Go back to more general specification where money demand depends on future inflation

$$\frac{M_t}{N_{1,t}P_t} = \frac{1}{2}\left(\alpha_1 - \alpha_2 \frac{P_{t+1}}{P_t}\right)$$

Iterating forward we get an expression for the price level

$$P_t = \frac{2}{\alpha_1}\sum_{j=0}^{\infty}\left(\frac{\alpha_2}{\alpha_1}\right)^j \frac{M_{t+j}}{N_{t+j}}$$

so the current price level (and inflation) depends on money today and in the future.
If future weights more than present (e.g. low T), then tight money now $\rightarrow$ high inflation now

This is the "tight money paradox" of Loyo (1999), and a provocative question for today's world

References

Loyo, Eduardo, "Tight Money Paradox on the Loose: A Fiscal Hyperinflation," 1999. Mimeo, Harvard University.

Sargent, Thomas J. and Neil Wallace, "Some unpleasant monetarist arithmetic," *Quarterly Review, Federal Reserve Bank of Minneapolis*, 1981, *Fall issue.*